

# Hyperfine Structure Constants for Eu Isotopes : Is The Empirical Formula of HFS Anomaly Universal ?

T. ASAGA<sup>1</sup>, T. FUJITA<sup>2</sup> and K. ITO<sup>3</sup>

Department of Physics, Faculty of Science and Technology

Nihon University, Tokyo, Japan

## ABSTRACT

We calculate the hyperfine structure constant for the Eu isotopes with shell model wave functions. The calculated results are compared with those predicted by the Moskowitz-Lombardi (M-L) empirical formula. It turns out that the two approaches give the very different behaviors of the hfs constants in the isotope dependence. This should be easily measured by experiment, which may lead to the universality check of the M-L formula.

PACS number: 21.10 Ky

---

<sup>1</sup>e-mail: [tasaga@shotgun.phys.cst.nihon-u.ac.jp](mailto:tasaga@shotgun.phys.cst.nihon-u.ac.jp)

<sup>2</sup>e-mail: [fffujita@phys.cst.nihon-u.ac.jp](mailto:fffujita@phys.cst.nihon-u.ac.jp)

<sup>3</sup>e-mail: [kito@shotgun.phys.cst.nihon-u.ac.jp](mailto:kito@shotgun.phys.cst.nihon-u.ac.jp)

## 1. Introduction

The magnetic hyperfine structure constant (hfs) has been extensively studied for various nuclei since it can present some interesting information on the magnetization distribution in nucleus [1-5]. The main difference between the magnetic moment operator and the magnetic hfs constant lies in the appearance of the new type of operator  $\Sigma_i^{(1)}$  as defined by

$$\Sigma_i^{(1)} \equiv s_i + \sqrt{2\pi}[sY^{(2)}]_i^{(1)}. \quad (1.1)$$

This operator  $\Sigma_i^{(1)}$  looks similar to the magnetic moment operator. However, it presents often quite different behaviors from the magnetic moment. In particular, this shows up in the isotope shifts of the hfs constant. For example, the isotope shifts of the hfs constant for the Mercury nuclei indicate that the expectation value of the  $\Sigma_i^{(1)}$  operator is constant over wide range of the shell model configurations.

Due to this fact, there is a remarkably good empirical formula which is proposed by Moskowitz and Lombardi (M-L formula) [5]. This M-L rule simply states that the isotope shifts of the hfs constant for the Hg nuclei can be well described if we write the hfs anomaly  $\epsilon$  as

$$\epsilon = \frac{\alpha}{\mu} \quad (1.2)$$

where  $\alpha = \pm 0.01$  n.m. for the valence neutrons with the spin of  $I = \ell \pm \frac{1}{2}$ , and  $\mu$  denotes the magnetic moment. The M-L formula can describe the isotope shifts  $\Delta_{12}$  of the hfs constant for the Hg nuclei surprisingly well.

Now, a question arises. Can the M-L formula work equally well for other nuclear isotopes as well? This is the main question we want to address in this paper.

Recently, Werth et al.[6] proposed to measure the isotope shifts  $\Delta_{12}$  of the hfs constant for the Eu nuclei. This is a very interesting isotope from the

point of view of the empirical formula. The measured magnetic moment of the Eu nuclei shows a drastic change at the mass number  $A = 153$ . For the isotopes of the Eu from  $A = 145$  to 151, the magnetic moment may well be described by the core polarization effects. However, the magnetic moments of  $A = 153$  and the heavier Eu nuclei are roughly one half of those of the lighter Eu isotopes.

This big change of the magnetic moments should show up if we use the empirical formula of eq.(1.1). Indeed, if we calculate the isotope shifts  $\Delta_{12}$  of the hfs constant for the Eu nuclei using the M-L empirical formula of eq.(1.2), then we obtain a sharp transition at  $A = 153$  for the  $\Delta_{12}$ .

On the other hand, we can also calculate the hfs constant of the Eu isotopes by using the shell model wave functions. This is what we have done in this paper. We calculate the magnetic hfs constant of the Eu nuclei by considering the core polarization effects. For the big change of the magnetic moment at  $A = 153$ , we consider some special state which is assumed to absorb the magnetic moment. This state must be connected to some collective state. But here we do not argue in detail which kind of properties this special state can possess, apart from the assumption that this state has a vanishing magnetic moment. Under this assumption, we obtain the isotope shifts  $\Delta_{12}$  for the Eu nuclei which do not show any sudden change at  $A = 153$ . This is mainly because the core polarization effects on the magnetic moment and on the  $\Sigma_i^{(1)}$  operators behave very similarly and thus they cancel with each other. This large difference between the two theories should be checked by experiment.

From the experimental side, there is an important progress to measure the isotope shifts of the magnetic hfs constant [6,7]. This becomes possible due to the ion trap method which can isolate the atoms. This ion trap method

can measure hfs separations with an accuracy of  $10^{-8}$  or even lower, which is by far better than the accuracy any theoretical models can predict. Also, the feasibility to measure directly nuclear magnetic moments by the ion trapping technique has been demonstrated successfully[8]. With this high accuracy of the ion trap method, there is some possibility to check the time reversal invariance in the atomic processes once nuclear ambiguities are removed. This is also one of our purpose to study the magnetic hfs constant, though we still do not have any good physical quantities at hand to study the time reversal invariance.

The paper is organized as follows. In the next section, we briefly explain the theory of the magnetic hyperfine structure in electronic atoms. Then, section 3 treats the calculation of the nuclear part of the hfs constant by using shell model wave functions. Also, we explain a special state which has a vanishing magnetic moment. In section 4, numerical results of the isotope shifts of the hfs constant for the Eu nuclei are presented. Also, the empirical formula of Moskowitz and Lombardi is compared to the present calculation. In section 5, we summarize what we have understood from this work.

## 2. Magnetic Hyperfine Structure Constant

The atomic electron which is bound by the nucleus feels the magnetic interaction in addition to the static Coulomb force. The magnetic interaction between the electron and the nucleus can be described as

$$H' = - \int \mathbf{j}_N(\mathbf{r}) \mathbf{A}(\mathbf{r}) d^3r \quad (2.1)$$

where the nuclear current  $\mathbf{j}_N(\mathbf{r})$  can be written as

$$\mathbf{j}_N(\mathbf{r}) = \frac{e\hbar}{2Mc} \sum_i g_s^{(i)} \nabla \times s_i \delta(\mathbf{r} - \mathbf{R}_i) + \sum_i g_\ell^{(i)} \frac{e}{2M} (\mathbf{P}_i \delta(\mathbf{r} - \mathbf{R}_i) + \delta(\mathbf{r} - \mathbf{R}_i) \mathbf{P}_i). \quad (2.2)$$

$\mathbf{A}(\mathbf{r})$  denotes the vector potential which is created by the atomic electron, and it can be written as

$$\mathbf{A}(\mathbf{r}) = \int \frac{\mathbf{j}_L(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3 r' \quad (2.3)$$

where  $\mathbf{j}_L(\mathbf{r})$  denotes the current density of the electron and is written as

$$\mathbf{j}_L(\mathbf{r}') = (-e)\alpha\delta(\mathbf{r} - \mathbf{r}').$$

In this case, the magnetic hyperfine splitting energy  $W$  can be written as

$$W = \langle IJ : FF | H' | IJ : FF \rangle = \frac{1}{2} [F(F+1) - I(I+1) - J(J+1)] a_I \quad (2.4)$$

where  $I, J$  and  $F$  denote the spin of the nucleus, the spin of the atomic electron and the total spin of the atomic system, respectively.  $a_I$  is called the magnetic hyperfine structure (hfs) constant. Following ref.[3], we can write the expression for the  $a_I$  as

$$a_I = a_I^{(0)}(1 + \epsilon) \quad (2.5)$$

where  $a_I^{(0)}$  is the hfs constant for the point charge, and can be written as

$$a_I^{(0)} = -\frac{2ek\mu_N}{IJ(J+1)} \mu \int_0^\infty F^{(kJ)} G^{(kJ)} dr \quad (2.6)$$

where  $F^{(kJ)}$  and  $G^{(kJ)}$  are the large and small components of the relativistic electron wave function for the  $kJ$  state.  $\mu_N$  is the nuclear magneton.

$\epsilon$  is called the hfs anomaly and can be written as

$$\epsilon = -\frac{1}{\mu} \langle II | \sum_{i=1}^A N(R_i) \mu_i | II \rangle - \frac{1}{\mu} \langle II | \sum_{i=1}^A K(R_i) g_s^{(i)} \Sigma_i^{(1)} | II \rangle \quad (2.7)$$

where  $\Sigma_i^{(1)}$  is defined in eq.(1.1).  $N(R)$  and  $K(R)$  are written for the atomic electron as,

$$N(R) = 0.62b^{(kJ)} \left( \frac{R}{R_0} \right)^2 \quad (2.8)$$

$$K(R) = 0.38b^{(kJ)} \left( \frac{R}{R_0} \right)^2 \quad (2.9)$$

where  $R_0$  is a nuclear radius and can be given as  $R_0 = r_0 A^{\frac{1}{3}}$  with  $r_0 = 1.2$  fm. On the other hand,  $b^{(kJ)}$  is a constant which can be calculated in terms of relativistic electron wave functions and can be written as [3].

$$b^{(kJ)} = 0.23k_0^2 R_0 \gamma (1 - 0.2\gamma^2) \left[ -\left(1 + \frac{4R_0 m_e c}{3\gamma \hbar}\right) \right] / \int_0^\infty F^{(kJ)} G^{(kJ)} dr. \quad (2.10)$$

$m_e$  denotes the electron mass,  $k_0^2$  is a normalization constant and the factor in the square bracket is to be included for  $p_{\frac{1}{2}}$  only, and  $\gamma = Ze^2/\hbar c$ .

The isotope shift of the hfs anomalies of the two isotopes  $\Delta_{12}$  is defined as

$$\Delta_{12} = \frac{a_{I_1} g_2}{a_{I_2} g_1} - 1. \quad (2.11)$$

Since the hfs anomaly  $\epsilon$  is quite small,  $\Delta_{12}$  becomes

$$\Delta_{12} \approx \epsilon_1 - \epsilon_2. \quad (2.12)$$

### 3. The hfs anomaly

The hfs anomaly  $\epsilon$  can be calculated if we know the nuclear wave function. However, it is often difficult and complicated to determine reliable nuclear wave functions.

Here, we employ simple-minded shell model wave functions with core polarizations taken into account. We take the following two approaches. The

first one is to consider only the  $\Delta\ell = 0$  core polarization for the  $\Sigma_i^{(1)}$  operator. In this case, we can calculate the matrix element of the  $\Sigma_i^{(1)}$  without introducing any free parameters as discussed in ref.[3]. On the other hand, if we want to include the  $\Delta\ell = 2$  core polarization for the  $\Sigma_i^{(1)}$  operator, then we should use the nuclear wave function which can be obtained by truncating the shell model spaces. In the case of the Eu nuclei, the contribution of the  $\Delta\ell = 2$  core polarization may be important since there are two orbits (  $2f_{7/2}$  and  $1h_{9/2}$  ) nearby which generate a large effect on the matrix element of  $[sY^{(2)}]_i^{(1)}$  operator.

(a)  $\Delta\ell = 0$  core polarization

First, we consider the  $\Delta\ell = 0$  core polarization for the  $\Sigma_i^{(1)}$  operator [9-11]. In this case, we can express the effect of the core polarization on the  $\Sigma_i^{(1)}$  operator in terms of the core polarization of the magnetic moment. This is mainly because the core polarizations of  $\sum g_s^{(i)} s_i$  and  $\sum g_s^{(i)} \sqrt{2\pi} [sY^{(2)}]_i^{(1)}$  operators are related as

$$\delta < \sum g_s^{(i)} \sqrt{2\pi} [sY^{(2)}]_i^{(1)} > = -\frac{1}{4} \delta < \sum g_s^{(i)} s_i > .$$

Therefore, we can write the expectation value of the  $\Sigma_i^{(1)}$  as

$$< II | \sum_{i=1}^A g_s^{(i)} \Sigma_i^{(1)} | II > = \pm g_s^{(VN)} \frac{3(I + \frac{1}{2})}{4(I + 1)} + \frac{3g_s^{(VN)}}{4(g_s - g_\ell)^{(VN)}} (\mu - \mu_{sp} - \delta\mu^{mes}) \quad (3.1)$$

for  $I = \ell \pm \frac{1}{2}$  as discussed in ref. [3]. Here,  $g_s^{(VN)}$  denotes the g-factor of the valence nucleon for the single particle state.  $\mu_{sp}$  is the single particle value of the magnetic moment operator and can be written as

$$\mu_{s.p.} = (I - \frac{1}{2})g_\ell^{(VN)} + \frac{1}{2}g_s^{(VN)} \quad \text{for } I = \ell + \frac{1}{2} \quad (3.2a)$$

$$\mu_{s.p.} = \frac{I}{I + 1} \left( (I + \frac{3}{2})g_\ell^{(VN)} - \frac{1}{2}g_s^{(VN)} \right) \quad \text{for } I = \ell - \frac{1}{2}. \quad (3.2b)$$

$g_s^{(VN)}$  and  $g_\ell^{(VN)}$  are taken to be the free nucleon g-factors,

$$g_\ell^{(p)} = 1.0, \quad g_s^{(p)} = 5.5855$$

$$g_\ell^{(n)} = 0, \quad g_s^{(n)} = -3.8263.$$

Also,  $\delta\mu^{mes}$  is the effective magnetic moment arising from the meson exchange current [12] and can be approximated by

$$\delta\mu^{mes} \approx 0.1\ell\tau_3. \quad (3.3)$$

In the present calculation, however, we do not take into account the exchange current effects. This is because it is not very easy to calculate

the  $\Delta\ell = 2$  core polarization effect together with the exchange current in a consistent way. In any case, the exchange current effects are not very large here.

Therefore, we do not have any free parameters in the evaluation of the expectation value of the  $\Sigma_i^{(1)}$  for the  $\Delta\ell = 0$  core polarization case.

(b)  $\Delta\ell = 2$  core polarization

The contribution of the  $\Delta\ell = 2$  core polarization to the operator  $\sum g_s^{(i)} \Sigma_i^{(1)}$  depends very much on the nuclear configurations. For example, there is little chance for light nuclei that the  $\Delta\ell = 2$  core polarization becomes important. However, in the Eu nuclei, there may be a large contribution from the  $\Delta\ell = 2$  core polarization. This can be easily seen if we look at the neutron configurations. For example, in the  $^{147}\text{Eu}$  nucleus, the two neutrons outside the  $N = 82$  magic shell may have the following orbits,  $2f_{7/2}$  and  $1h_{9/2}$  nearby which have almost the same single particle energies. Therefore, there is a strong mixture between them due to the  $[sY^{(2)}]^{(1)}$  operator, and thus we have to consider these configurations carefully.

Since the Eu isotopes with even number of neutrons have the spin of  $I = \frac{5}{2}$ , the proton state may be described by  $2d_{5/2}^{-1}$ . Also, the neutron number  $N = 82$  is a magic shell, and therefore the Eu nucleus with the mass number  $A = 63 + 82 + n$  should have the  $n$  neutrons outside the  $N = 82$  shell. Therefore, the wave function for the Eu nucleus with  $n$  neutrons may be constructed by the following three states

$$|\Psi_0 : n \rangle = \alpha_1 |1 \rangle + \alpha_2 |2 \rangle + \alpha_3 |3 \rangle \quad (3.4)$$

where

$$|1 \rangle = |\pi(2d_{5/2})^{-1}, \nu(2f_{7/2})^{(n)}(0^+) : II \rangle \quad (3.5a)$$

$$|2\rangle = |\pi(2d_{\frac{5}{2}})^{-1}, \nu \left( (2f_{\frac{7}{2}})^{(n-1)} 1h_{\frac{9}{2}} \right)_{(1+)} : II\rangle \quad (3.5b)$$

$$|3\rangle = |\pi(2d_{\frac{5}{2}})^{-1}, \nu \left( (2f_{\frac{7}{2}})^{(n-1)} 2f_{\frac{5}{2}} \right)_{(1+)} : II\rangle. \quad (3.5c)$$

$\alpha_1, \alpha_2$  and  $\alpha_3$  should be determined by the nuclear residual interaction.

Note that  $I$  is here  $\frac{5}{2}$ .

To determine the values of the  $\alpha_i$ , we employ the  $\delta$ -function force for the residual interaction for simplicity [11],

$$V_{12} = -V_0 \delta(\mathbf{r}_1 - \mathbf{r}_2). \quad (3.6)$$

Now, the problem is that we cannot use the perturbation theory here since the unperturbed energies of the state  $|1\rangle$  and the state  $|2\rangle$  are degenerate. Therefore, we should diagonalize the hamiltonian with the residual interaction. Denoting the unperturbed energies for the states  $|i\rangle$  as  $E_i$ , we obtain the matrix equations which determine the values of  $\alpha_i$ .

$$(E_1 + V_{11} - E)\alpha_1 + V_{12}\alpha_2 + V_{13}\alpha_3 = 0 \quad (3.7a)$$

$$V_{21}\alpha_1 + (E_2 + V_{22} - E)\alpha_2 + V_{23}\alpha_3 = 0 \quad (3.7b)$$

$$V_{31}\alpha_1 + V_{32}\alpha_2 + (E_3 + V_{33} - E)\alpha_3 = 0. \quad (3.7c)$$

Here, we can take  $E_1 = E_2$  to a good approximation. Also, we can neglect the interference term between the  $|2\rangle$  and  $|3\rangle$  states.

Before going to determine the values of the parameters  $V_0$  and  $E_1$ , we discuss the core polarizations which contribute to the magnetic moment. Since the N=82 is the magic shell, the neutrons of the  $^{145}\text{Eu}$  nucleus are all filled. In this case, there is no contribution from the  $\Delta\ell = 2$  core polarization. Instead, the  $\Delta\ell = 0$  core polarization comes from the neutrons configuration mixing of

$$|\pi(2d_{\frac{5}{2}})^{-1}, \nu \left( (1h_{\frac{11}{2}})^{-1} 1h_{\frac{9}{2}} \right)_{(1+)} : II\rangle$$

state, and the proton configuration mixing of

$$|\pi(2d_{\frac{5}{2}})^{-2}, (2d_{\frac{5}{2}}) : II >$$

state. We take these effects perturbatively.

For the heavier Eu isotopes, we assume that this part of the  $\Delta\ell = 0$  core polarization behaves just in the same way as the  $^{145}\text{Eu}$  nucleus. Therefore, the nuclear state of eq.(3.4) should reproduce the magnetic moment which is the observed magnetic moment plus the contribution from the  $\Delta\ell = 0$  core polarization. This can be easily obtained since the  $^{145}\text{Eu}$  has only the  $\Delta\ell = 0$  core polarization, which is  $\delta\mu_{CP} = -0.8$  n.m.

However, this is possible only up to the  $A = 151$  nucleus, and there is a big change of the observed magnetic moment from  $A = 151$  to  $A = 153$ . The magnetic moment of  $A = 153$  nucleus is smaller than  $A = 151$  by a factor 2. This means that we cannot reproduce the magnetic moment of the nuclei heavier than the  $A = 153$  by the simple-minded core polarization.

Therefore, we should consider some kind of collective state (deformed state) into the original state  $|\Psi_0 : n >$ . We call this new state  $|MAS >$  state (magnetic absorbing state) since the  $|MAS >$  state is assumed to have a vanishing magnetic moment. That is,

$$< MAS | \sum_{i=1}^A \mu_i | MAS > = 0. \quad (3.8a)$$

At the same time, we assume that the expectation value of the operator  $\sum g_s^{(i)} \Sigma_i^{(1)}$  with the MAS state vanishes,

$$< MAS | \sum_{i=1}^A g_s^{(i)} \Sigma_i^{(1)} | MAS > = 0. \quad (3.8b)$$

In this case, we can construct the wave functions for the  $^{153}\text{Eu}$  nuclei.

We write

$$|\Psi : MAS \rangle = \sqrt{1 - \alpha_4^2} |\Psi_0 : n = 8 \rangle + \alpha_4 |MAS \rangle. \quad (3.9)$$

Here, the value of the parameter  $\alpha_4$  can be determined such that the observed magnetic moment can be reproduced for the  $^{153}\text{Eu}$  nuclei.

Now, concerning the configurations for the  $A=155$  and heavier nuclei, the  $2f_{7/2}$  states are filled. Therefore, we take the following configuration

$$|\widetilde{\Psi}_0 : n \rangle = \alpha_1 |\widetilde{1} \rangle + \alpha_2 |\widetilde{2} \rangle + \alpha_3 |\widetilde{3} \rangle \quad (3.10)$$

where

$$|\widetilde{1} \rangle = |\pi(2d_{5/2})^{-1}, \nu(1h_{9/2})^{(n-8)}(0^+) : II \rangle \quad (3.11a)$$

$$|\widetilde{2} \rangle = |\pi(2d_{5/2})^{-1}, \nu((1h_{9/2})^{(n-7)}(2f_{7/2})^{-1})_{(1^+)} : II \rangle \quad (3.11b)$$

$$|\widetilde{3} \rangle = |\pi(2d_{5/2})^{-1}, \nu((2f_{7/2})^{-1}2f_{5/2})_{(1^+)} : II \rangle. \quad (3.11c)$$

Here, the value of the  $\alpha_3$  is fixed to the one determined for  $A=153$ .

Further, the wave function is taken to be

$$|\Psi : MAS \rangle = \sqrt{1 - \alpha_4^2} |\widetilde{\Psi}_0 : n \rangle + \alpha_4 |MAS \rangle. \quad (3.12)$$

These are the shell model wave functions which we use for our calculations. However, they are obviously too simple-minded wave functions, but we believe that we can get some idea as to what are the contributions from the  $\Delta\ell = 2$  core polarization as well as from the sharp change of the magnetic moment to the hfs constant in the Eu isotopes.

### (c) Moskowitz-Lombardi empirical formula

Moskowitz and Lombardi proposed an empirical formula in order to explain the isotope shifts of the hfs anomaly in Hg isotopes [5]. They

simply take the shape of the hfs anomaly to be

$$\epsilon = \frac{\alpha}{\mu}$$

where  $\alpha$  is a constant. When they take  $\alpha$  to be  $\pm 0.01$  n.m. for  $j = \ell \pm \frac{1}{2}$  neutron orbits, they obtain a remarkably good description of the isotope shifts  $\Delta_{12}$  for the Hg nuclei.

Here, we want to apply this formula to the Eu isotopes. In this case, we should make a correction due to the atomic orbit. It should reflect in the value of the  $\alpha$  since the hfs anomaly depends on  $b^{(kJ)}$ . Therefore, we make this atomic correction as

$$C = \frac{b^{(kJ)}(Eu)}{b^{(kJ)}(Hg)} \approx 0.5.$$

Therefore, we take the  $\alpha$  to be  $\mp 0.005$  n.m. for  $j = \ell \pm \frac{1}{2}$  proton orbits.

#### 4. Numerical Results

Once we know the values of  $\alpha_i$  in eqs.(3.5) and (3.11), then we can calculate the expectation values of the magnetic moment as well as the  $\Sigma_i^{(1)}$  operator.

Before going to the numerical calculations, we make the valence nucleon approximation to the expectation values of  $\langle (\frac{R_i}{R_0})^2 \rangle$ . Since the dominant contributions to the magnetic moment as well as to the  $\Sigma_i^{(1)}$  operator come from the valence nucleons, it is always a good approximation to factorize the  $\langle (\frac{R_i}{R_0})^2 \rangle$  and  $\mu_i$  or  $\Sigma_i^{(1)}$ . Therefore, eq.(2.7) can be written as

$$\epsilon = -0.62b^{(kJ)} \langle \left(\frac{R_i}{R_0}\right)^2 \rangle_{VN} - 0.38b^{(kJ)} \langle \left(\frac{R_i}{R_0}\right)^2 \rangle_{VN} \frac{1}{\mu} \langle II | \sum_{i=1}^A g_s^{(i)} \Sigma_i^{(1)} | II \rangle \quad (4.1)$$

where  $\langle (\frac{R_i}{R_0})^2 \rangle_{VN}$  is the expectation value with the valence nucleons.

Now, we can calculate the hfs anomaly with the shell model wave functions. First, we consider the  $\Delta\ell = 0$  core polarizations. In this case, we do not have any free parameters since all the terms in eq.(3.1) are known. In Table 1, we show our calculated results of the hfs anomaly with the  $\Delta\ell = 0$  core polarization for the Eu nuclei. Also, the calculated results of the isotope shifts  $\Delta_{145,A}$  are shown.

Next, we consider the  $\Delta\ell = 2$  core polarizations. In this case, we should first determine the values of  $\alpha_i$ . Basically we change the value of the  $E_3$  so that we can obtain the observed magnetic moment.

In Table 2, we show our calculated results of the hfs anomaly including the  $\Delta\ell = 2$  core polarizations. Also, the isotope shifts of the hfs anomaly are shown. Here, as mentioned before, we have not considered the exchange current effect on the hfs anomaly. Therefore, there is no difference in the hfs anomaly for the  $^{145}\text{Eu}$  between Table 1 and Table 2 since  $^{145}\text{Eu}$  does not have any effects due to the  $\Delta\ell = 2$  core polarizations. On the other hand, the difference in the hfs anomaly between Table 1 and Table 2 for the heavier Eu isotopes is partly due to the  $\Delta\ell = 2$  core polarization effect and partly due to the MAS state. In particular, the large difference between the two calculations for the Eu nuclei heavier than the  $A=153$  comes from the MAS state. The  $\Delta\ell = 2$  core polarization contribution to the hfs anomaly for the  $^{147}\text{Eu}$  is about  $\epsilon^{\Delta\ell=2} \approx 0.01$  %.

These calculations should be compared to the prediction by Moskowitz-Lombardi empirical rule of eq.(1.2). Note that the large difference of the hfs anomaly  $\epsilon$  in magnitude between the present calculations and the M-L formula is mainly due to the first term in eq.(4.1). However, the first term in eq.(4.1) contributes very little to the isotope shifts  $\Delta_{12}$  since it is

practically constant.

In order to see more clearly the difference between the models, we plot in fig. 1 the isotope shifts of the hfs anomaly as the function of the nuclear mass number  $A$ .

First, we want to present the results of the Mercury isotopes so that we can obtain some idea how these models can describe the data. In fig. 1a, we show the predictions of the  $\Delta_{199,A}$  for the Hg nuclei by the different models. There, the difference between the model calculations is not very large. At least, there is a qualitative agreement in the behavior, although the M-L formula (dashed line) can describe the data better than the shell model calculations. The FULL (solid line) in fig. 1a indicates that the calculation includes both the  $\Delta\ell = 0$  and  $\Delta\ell = 2$  core polarizations. Here, however, the  $\Delta\ell = 2$  core polarization is not very large [3].

In fig.1b, we show the calculated results for the Eu nuclei. In this case, there is a big difference between the model calculations of the isotope shifts  $\Delta_{145,A}$ . The Moskowitz-Lombardi empirical formula (dashed line) predicts a big change in the isotope shifts  $\Delta_{145,A}$  at the  $A = 153$  nucleus. On the other hand, our theoretical estimations show a very different behavior of the isotope shift  $\Delta_{145,A}$ . In particular, the predictions with the  $\Delta\ell = 0$  core polarizations (dotted line) give a completely opposite behavior to the M-L result. The sign of the isotope shifts  $\Delta_{145,A}$  are negative. Also, the FULL shell model calculations (solid line) show a quite smooth transition at  $A=153$ . This is mainly due to the introduction of the MAS state.

Therefore, it should be extremely interesting to check experimentally which of the models can be reasonable. Further, it would be very interesting to understand whether there is any universality in the M-L formula.

## 5. Conclusions

We have presented the numerical calculations of the isotope shifts of the magnetic hfs anomaly for the Eu nuclei. Here, we have considered the  $\Delta\ell = 0$  as well as the  $\Delta\ell = 2$  core polarizations to the hfs operators. It turns out that the calculations with both of the core polarizations do not show any sharp transition at  $A = 153$  where the observed magnetic moment show a big change. This is because the hfs operators and the magnetic moment operators behave similarly by the core polarizations, and therefore the effects of the core polarizations cancel with each other.

On the other hand, there is a very nice empirical formula by Moskowitz-Lombardi, which perfectly describes the isotope shifts of the hfs anomaly in Hg nuclei. Here, we also employ the M-L formula how it predicts the isotope shifts of the hfs anomaly in the Eu nuclei. It turns out that the M-L formula predicts a transition at  $A = 153$  since this is essentially proportional to the inverse of the magnetic moment.

Therefore, it would be extremely nice to learn which of the pictures the nature prefers. Also, it is quite interesting to understand if there is any universality in this simple M-L rule.

Acknowledgment: We thank I. Katayama and G. Werth for discussions. This work is supported in part by Japanese-German Cooperative Science Promotion Program.

## References

1. A. Bohr and V.F. Weisskopf, Phys. Rev. **77**, 94 (1950)
2. J. Johnson and R.A. Sorensen, Phys. Rev. **C2**, 102 (1970)
3. T. Fujita and A. Arima, Nucl. Phys. **A254**, 513 (1975)
4. S. Buttgenbach, Hyperfine Structure in 4d and 5d-Shell Atoms,  
Springer Tracts in Modern Physics **96** (1982)
5. P.A. Moskowitz and M. Lombardi, Phys. Lett. **46B**, 334 (1973)
6. G. Werth, S. Trapp, G. Bollen, S. Schwarz and A.Kohl, CERN/ISC 96-9
7. M. Wada, K. Okada, H. Wang, K. Enders, F. Kurth, T. Nakamura, S. Fujitaka,  
J. Tanaka, H. Kawakami, S. Ohtani and I. Katayama, to appear in Nucl. Phys. **A**
8. K. Enders, O. Becker, L. Brand, J. Dembczynski, G. Marz, G. Revalde,  
P.B. Rao and G. Werth, Phys. Rev. **A52**, 4434 (1995)
9. A. Arima and H. Horie, Prog. Theor. Phys. **12**, 623 (1954)
10. R.J. Blin-Stoyle and M.A. Perks, Proc. Phys. Soc. **A67**, 885 (1954)
11. H. Noya, A. Arima and H. Horie, Prog. Theor. Phys. Suppl. **8**, 33 (1958)
12. S. Nagamiya and T. Yamazaki, Phys. Rev. **C4**, 1961 (1971)
13. R.L. King, C.H. Liu, G. Fulop and H.H. Stroke, Phys. Lett. **B31**, 567  
(1970)
14. P.A. Moskowitz, C.H. Liu, G. Fulop and H.H. Stroke, Phys. Rev. **C4**, 620  
(1971)

Table 1

The hfs anomaly for Eu isotopes  $(\Delta\ell = 0)$

A	$\mu$	$b^{(1s\frac{1}{2})}$ (%)	$-\epsilon$ (%)	$\Delta_{145,A}$ (%)
145	3.993	1.58	1.067	0
147	3.724	1.58	1.056	-0.011
149	3.565	1.59	1.053	-0.014
151	3.472	1.59	1.046	-0.021
153	1.533	1.60	0.919	-0.148
155	1.56	1.60	0.919	-0.148
157	1.5	1.61	0.912	-0.155
159	1.38	1.62	0.893	-0.174

We plot the hfs anomaly and the isotope shifts for the Eu isotopes with the  $\Delta\ell = 0$  core polarization. In addition, we show the magnetic moment and the electron wave function coefficient  $b^{(k_J)}$  for  $1s_{\frac{1}{2}}$ .

Table 2

The hfs anomaly for Eu isotopes  $(\Delta\ell = 0) + (\Delta\ell = 2)$

A	$\mu$	$b^{(1s\frac{1}{2})}$ (%)	$-\epsilon$ (%)	$\Delta_{145,A}$ (%)	$-\epsilon^{ML}$ (%)	$\Delta_{145,A}^{ML}$ (%)
145	3.993	1.58	1.067	0	0.125	0
147	3.724	1.58	1.044	-0.023	0.1349	0.009
149	3.565	1.59	1.042	-0.025	0.140	0.015
151	3.472	1.59	1.036	-0.031	0.144	0.019
153	1.533	1.60	1.039	-0.028	0.326	0.201
155	1.56	1.60	1.035	-0.032	0.32	0.20
157	1.5	1.61	1.019	-0.048	0.33	0.21
159	1.38	1.62	1.019	-0.048	0.36	0.24

We plot the hfs anomaly and the isotope shifts for the Eu isotopes with the  $\Delta\ell = 0$  and  $\Delta\ell = 2$  core polarizations. Also, we show the predictions by the M-L empirical formula with the universal coupling of  $\alpha = -0.005$  n.m.

## Figure Captions :

Fig.1a :

The isotope shifts  $\Delta_{199,A}$  of the hfs anomaly for the Hg nuclei are shown as the function of the mass number. The solid line (FULL) denotes the shell model calculation including both the  $\Delta\ell = 0$  and  $\Delta\ell = 2$  core polarizations. The dashed line (M-L) is the calculation by the M-L empirical formula (  $\epsilon^{ML}$  and  $\Delta_{A,145}^{ML}$  ). The black circles denote the experiment [13,14].

Fig.1b :

The isotope shifts  $\Delta_{145,A}$  of the hfs anomaly for the Eu nuclei are shown. The same as fig.1a for the solid and dashed lines. The dotted line denotes the calculation only with the  $\Delta\ell = 0$  core polarization.